CURVILINEAR MORPHO-HESSIAN FILTER

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ABSTRACT

The motivation of this paper is the filtering of thin elongated objects, such as veins, fibres etc. In particular, we focus our attention on detecting thin segments and linking them when disconnection is due to noise. A hybrid Hessian-based and morphological linear filtering method is proposed within the framework of scale space theory. For each pixel the second-derivative response kernel is collected and formed into a Hessian matrix which undergoes eigen analysis. From there, the best eigenvalue response along with the corresponding eigenvector is chosen on different scales. Using the resulting principal directions of linear segment pixels, morphological filters are used to track and connect the linears.

***Index Terms*—** linear filter, vesselness, morphological operations, opening, Hessian-based filtering, directional filtering.

1. INTRODUCTION

There exists a great range of applications from industrial to biomedical fields which require from image analysis a segmentation of narrow, curvilinear objects. Those frequently studied thin objects can be solid materials fissures, blood vessels, natural or artificial fibres, fingerprint lines and many others. The segmentation interest can lie in a whole object representation (e.g. fiber surface reconstruction) or in a certain characteristic of it (e.g. topology, size, curvature). Moreover, extraction of linear objects and segments is an essential step for the segmentation of whole line networks and/or parts of larger objects (centrelines such as skeletons).

However, before confronting an object segmentation question, even despite nowadays high resolution, precision and overall quality digital imaging, there is a need for image improvement. In order to improve linear object, a detection of such features is appropriate. Many narrow-object detection/segmentation methods use strong shape constraints, which are not appropriate for a wide range of problems with varying shape, size of line segments as well as the topology of their networks. From another point of view, intensity information alone is rarely enough due to original data and/or the nature of objects. In cases of vessels, fibers and many others, narrow objects intensity is not homogeneous. As a result, the objects may appear as thin disconnected filaments. The above are the motivations for a homogenizing and linking filtering method.

In many recent vessel-segmentation works, a multiscale set of Gaussian filters with a shape prior was used ([1, 2]). These works have shown that the method works well in detection and tracking of curvilines when the knowledge of searched objects is explored. In those, size and orientations of linear structures can be identified by derivative responses on multiple scales combined with Gaussian smoothing. As an extension, to support a wider range of object sizes and quality of SNR in the images, a multiscale approach as well as its optimal single scale one have been introduced. In the recent work of Manniesing [3], a diffusion filter is used in combination with the vesselness filtering and detection method by Frangi [1].

On the other hand, mathematical morphology researchers have studied linear object extraction, extending it to the direction detection and tracking [4, 5, 6, 7]. Gray-scale mathematical morphology operations have been shown to exhibit good results at local filtering, connection and disconnection of objects. However, to our knowledge, the possibility of combining Hessian-based object filtering with mathematical morphology methods has not yet been explored .

In the current study, we propose a combined enhancement method composed of Hessian-based and morpological filters. A matched filter is composed with directional second-degree derivatives of Gaussian kernels resulting in a Hessian matrix. This is done in order to track linear objects while enhancing them and interconnecting with morphological operations.

The paper is organised as follows: section 2 introduces Hessian-based filter, as well as its multiscale approach and vesselness filter. In section 3 a grey-scale morphologcal filtering is described. Experimental resuls are presented in section 4. The article is concluded in section 5.

2D Shapes		
λ_1	λ_2	
Big+	Small	line(dark)
Big-	Small	line (bright)
Big+	Big+	blob (dark)
Small-	Small-	blob (bright)

Table 1. Possible eigenvalue responses and their signs after the eigen analysis corresponding to different shapes and color intensities.

2. HESSIAN-BASED LINE DETECTION

In order to detect linear sructures in the image, the second derivative with a Gaussian kernel at scale σ produces a response of local intensities showing the contrast inside and outside the scale range in the direction of the derivative. This reponse kernel can be represented by the 2D Hessian matrix:

$$\mathbf{H} = \begin{bmatrix} F_{xx} & F_{xy} \\ F_{xy} & F_{yy} \end{bmatrix}, \tag{1}$$

where its elements are the second partial derivatives of an image *F(x, y)*, which are obtained by the convolution of the Gaussian kernel at scale σ. The Hessian matrix is processed through eigen analysis in order to extract the principal directions in which the second order object representation can be decomposed. Let λ_1 and λ_2 ($|\lambda_1| \geq |\lambda_2|$) be the eigenvalues of **H** and e_1, e_2 their corresponding eigenvectors, respectively.

The direction along which the function *F* has the maximal oriented second-derivative eigenvalue λ_1 (the minimal intensity variation) is eigenvector e_1 (considering that the studied object is darker than the background). The eigenvector e_2 is orthogonal to the vector e_1.

To summarize, in the 1 the relations of Hessian eigenvalues are shown for different shapes and intensities for 2D images.

2.1. Vesselness

Based on these relations, linear objects can be detected as well as enhanced. Frangi et al. have [1] introduced a vesselness measure function which identifies and enhances vessels and for 2D images can be expressed as (for dark objects on a light background):

$$\nu(x,\sigma) = \begin{cases} \mathit{0} & \mathit{if}\ \lambda_1 < 0, \\ \exp(-\frac{R_B^2}{2\beta^2})(1-\exp(\frac{-S^2}{2c^2})) & \mathit{otherwise.} \end{cases} \tag{2}$$

$$\begin{aligned} R_B &= \tfrac{\lambda_1}{\lambda_2}, \\ S &= \|H_\sigma\| = \sqrt{\textstyle\sum_j \lambda_j^2}, \end{aligned} \tag{3}$$

where R_B differentiates lines from blobs by considering the eccentricity of the second-order ellipse. S is the Frobenius matrix norm to differentiate objects of interest from the background, regarding the background having low second derivative responses. The parameters β and c are weighting factors determining the influence of R_B and S. The result of this filter yields the probability of a pixel being a linear object. The response is higher in the center of the object and decays slowly toward the boundaries.

As stated in the Equation 2, the filter can be applied at different scales, which can provide a result in a larger range of line sizes. The vesselness function is normalized by σ^2 [8] and the maximal vesselness is selected for each pixel

$$V(x) = max_{\sigma_{min} \leq \sigma \leq \sigma_{max}} \nu(x,\sigma)$$

However, the filter performance depends on how well the scale is chosen.

3. MORPHOLOGICAL FILTERING

3.1. Existing work

Basic filters in the mathematical morphology framework are not auto-dual, one needs to distinguish dark objects over bright backgrounds and their converse. In summary and in broad terms, orientation of a dark object on a bright background in 2D can be estimated at any point, given a segment structuring element (SE) of fixed length L, by computing the angle at which the corresponding closing is the darkest.

$$\mathit{orientation}(x,y) = \mathit{argmin}_\alpha\{\phi_{\alpha,L}[I](x,y)\}, \tag{4}$$

where I is the image, L is the length of the segment SE, and ϕ is the standard notation for a closing. A similar method holds for dual bright objects on dark backgrounds using openings instead. This method can also readily be extended to 3D and can feature multiscale analysis by varying L. Furthermore, a notion of *oriented object* can be defined by studying objects which have varying closing response when α varies. Intuitively, non-oriented objects have constant closing whatever the orientation, while thin objects, for instance, have one preferred orientation.

Filtering using oriented SE is classically performed, for dark objects, using infimums of closings at arbitrary directions using segments or paths structuring elements. For bright objects, supremums of openings are used instead.

3.2. Proposed method

Using infimums of closings for dark objects tends to preserve thin elongated objects while removing dark compact noise. However, curvilinear segments cannot be reconnected using this method, as a closing is extensive by definition and a dark object reconnection is anti-extensive. An anti-extensive opening must be used instead. However, an opening using an isotropic SE is likely to destroy the thin structures in the image. Hence, we must use an oriented opening.

In this work, we use the orientation information of the Hessian to drive morphological operations on account of its quality in linear objects direction detection, efficiency and multi-scale approach.

A first approach could be to filter the dark linear objects by an opening with a segment oriented in the same local direction as the object itself. The resulting operation ψ^o would be anti-extensive (ψ^o(image) $\leq$ image), since at each point it is the result of an opening, as well as increasing (if image1 $\geq$ image2, then ψ^o(image1) $\geq$ ψ^o(image2)) for the same reason. However it would not be idempotent. Indeed, iterating the operator ψ^o would yield successively different results, with no guarantee of convergence. The operator ψ^o is therefore not an opening, which converges in one step. These classical properties of morphological filters are important for practitioners because they allow for easier and better-behaved operators combinations. Instead, we propose the following algebraic opening using structuring functions rather than structuring elements, i.e. a structuring element $B(x)$ which varies with position x.

Given the dilation $\delta_{B(x)}[I](x) = \bigvee_{b \in B(x)} I(x+b)$, the adjunct erosion ε^* is non-local, i.e. the

$$\begin{aligned} &\forall x \in I, \forall (y,b), y \in I, b \in B(y) \text{ and } x = y + b, \\ &\varepsilon^*[I](x) = \textstyle\bigwedge_y f(x) \end{aligned} \quad (5)$$

This non-local definition is not very helpful, since it does not give rise to an efficient algorithm. However, we can shift the point of view from the set of points x to the set of points y. The definition becomes :

$$\forall x \in I, \forall b \in B(x),\ \varepsilon^*[f](b) = \bigwedge (f(x), f(x+b)) \quad (6)$$

The two definitions are equivalent, and the second yields a useful algorithm: for all points x of I, we propagate the value of x over all the points of the structuring element $B(x)$ that are of lower value. This algorithm has complexity $O(N \times L)$, with L the number of points in the structuring element and N the number of points in the image. The opening is consequently:

$$\gamma^o[I](x) = \delta_{B(x)}(\varepsilon^*_{B(x)})[I](x) \quad (7)$$

Here $B(x)$ is a variable structuring element. We used a centered line segment oriented in the direction α, which is provided by the principal direction from the Hessian computation. Length L can be variable at no extra cost, e.g. proportional to the principal value, however this has not been tested as of this writing.

This structured-function opening allows dark structures to extend in the direction of the Hessian principal direction, in turn permitting curvilinear segments reconnections. When orientation is unclear, $B(x)$ is a single point, which limits side effects.

4. EXPERIMENTS

The experimentation platform was written with the image analysis library ITK [9], using an implementations of vessel detection methods by [10] and [11] for comparison.

Each image pixel is processed by the second-derivative filter resulting in an Hessian matrix in its eigen form. From it, we choose the eigenvalue appropriate to a linear object according to Table 1 and the corresponding eigenvector (i.e. the principal line direction). Following the direction of the detected line segment, the local neighborhood of the original image is opened by the proposed structuring function, using $L = 11$ and $\alpha = \arg(e_1) \forall (x,y) \in I$. For each pixel, the best scale response of an eigenvalue and corresponding to its eigenvector are chosen. For the following neurite image (Figure 1), a range scale of 0.25-1 pixels in four steps was chosen. Our filtering results are compared to the ones of Frangi's vesselness with the same scale parameters and vesselness parameters $\beta = 0.5$ and $c = 5.0$.

An extract of a neurite image is shown in Figure 1(a). From fig. 1(d), we find that the proposed filter acts as a reconnecting filter as designed, without disturbing the fine structure of the vessels. Arrows in the original and final filtered image indicate some areas where reconnections occurred. In contrast, from fig. 1(c) we note that the max eigenvalue response does not have this effect. In fig. 1(c), the vesselness response filter exhibits poor performances, which is typical of its action on narrow and noisy blood vessels.

In summary, the mixed Hessian-morphological filter is acting in a homogenizing and connecting manner on the linears, while keeping intact dark background, non-linear objects. Since it is an opening, it also removes short, compact white areas, which is not evident in this example. This is usually desirable or of no consequence for dark features.

5. CONCLUSION

A non-linear, mixed Hessian-based and morphological method has been proposed in order to enhance linear objects within the scale-space theory. Preliminary tests on neurite images show that the proposed filtering scheme is giving encouraging results. The results are especially good for narrower linears, and this is where most existing filters fail, either linear or morphological.

The proposed filter has all the properties of a morphological filter (opening or closing), which ensures it can be used within a morphological processing pipeline with no side effect.

Our future interest lies in extensive testing, especially of closely placed to each other objects, cross-sections, as well as in distinguishing and enhancing other kind of shapes than linear. In addition, it is envisaged to use a variable and more flexible structuring element for morphological directional operations, such as paths instead of segments. Our other ambi-

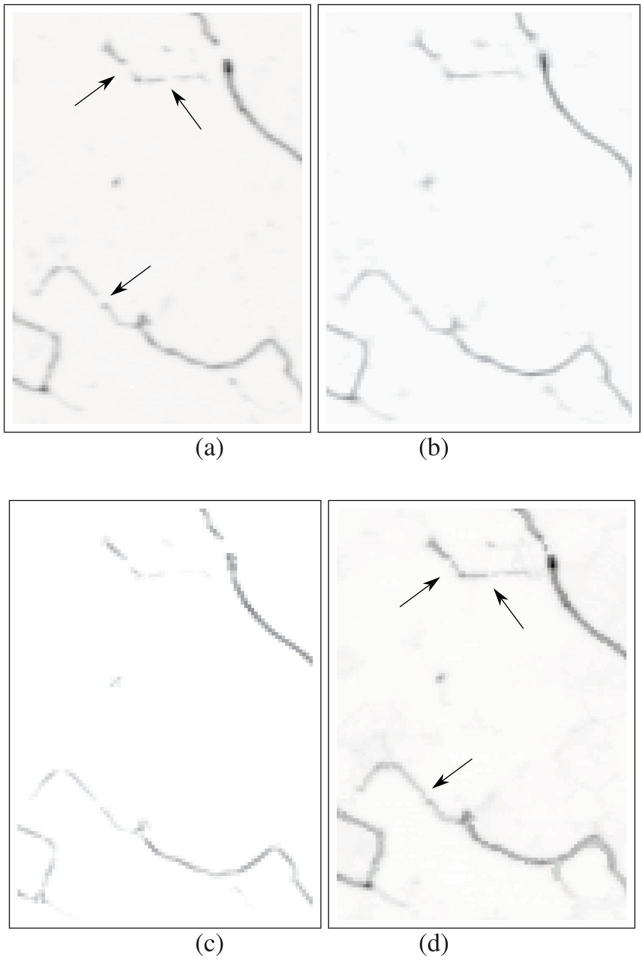

Fig. 1. Filtering example on an image of a neurite. (a) Original, (b) max Hessian eigenvalue, (c) vesselness response, (d) proposed filter. Arrows indicate areas of reconnection.

tion is in the extension of the proposed method for 3D images. Among all of these, we are interested in quantifying our filtering results and comparing them with other curvilinear filters.

Finally we plan to propose an improved algorithm with less memory consumption than the straightforward implementation. The current implementation is sufficient for work on 2D images, however.

6. ACKNOWLEDGEMENTS

The authors would like to thank Nicolas Passat and Michel Couprie for their generous insights.

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